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INTERPRETATION VERSUS DECISION OR THE TRUE FUNCTION OF THE SIGN--ETC(U)

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INTERPRETATION versus DECISION
OR
THE TRUE FUNCTION OF THE SIGNAL PROCESSOR DELINEATED

S. E. Smith

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I. INTRODUCTION

The procedure usually called "signal processing" may be factored into two parts:

(a) data interpretation and
(b) decision making.

It is the contention and thesis of this paper that only the former is the proper realm of the signal processor; that decision making is a line or command function while signal processing is an interpretive or staff function; and that confusion, misunderstanding, and inefficient system design result when these two separable notions are mingled and confounded.

It is not my position that signal processors should never make decisions, nor do I claim that tactical or line commanders should not engage in signal interpretation. However, I do insist that (these functions being separate) those who engage in both should know at each moment which role they then are playing.

The deep-seated confusion about this central issue is reflected in the naive terminology which pervades the world of the signal processor-user. For instance:

Detection: At some times this means "rectification"; at others it means that a voltage has exceeded a "preset" (but probably unspecified) threshold; at yet others it means that a target (of some probably agreed upon sort, probably unspecified) has been "observed" and "recognized." A roomful of signal processors and users will probably, in a 10-minute period, use the word "detection" in at least two of these three senses, with at least a 50% chance of misunderstanding.

False alarm: This means ordinarily that a "detection" [in the second sense above] has occurred when the user of the word "false alarm" wishes it hadn't. The tendency for users to set the words "detection" and "false alarm" in opposition shows the extent of the terminological blur. Actually, of course, "false alarms" are a subset of "detections."

The fact is that received data--in the form of radio messages, sonar or radio echoes, or whatever--serve to revise our concept of the probabilities which describe what we believe about the world. It is the signal processor's job to assess how probable it is that we would get the data--messages, echoes, or whatnot--which we have actually gotten, subject to all the various possible exclusive alternate hypotheses about what may be true.

The next step is to combine these results with a pre-message probabilistic description of the situation, and so to produce revised, updated, and improved descriptions. These revised descriptions (in the form of probabilities) then enable those concerned with decision making to make the best decisions possible on the basis of available information.

For instance, consider the following idealized and simplified problem: Suppose there is one target only and that there are n locations at which this target may be. Suppose that, after processing all signals available, an ideal processor concludes that the probability the target is not present at all is ρ_0, the probability the target is present and is in location #1 is ρ_1, the probability the target is in location #2 is ρ_2, etc. The ρ vector can be used as input to many tactical problems. Suppose the target is of interest to us only in that it presents a threat to ourselves. Suppose we wish to attain, as a minimum, a 98% probability of survival. Suppose we have devices (probably expensive) with which we can destroy the target at any given location if we choose and if it is really there. Then, clearly, if ρ_0 exceeds .98, we need expend no ammunition under these rules. If $\rho_0 < .98$, we note that

$$\rho_0 = 1 - \left[\rho_1 + \rho_2 + \dots + \rho_n\right] ,$$

and we attack the location with the largest ρ_j. If the elimination of ρ_j is sufficient to raise ρ_0 above .98, attacking one location is sufficient. If not, we attack the next largest ρ as well, etc., until the resultant ρ_0 exceeds .98.

By this procedure we attain a survival probability ≥ 98% while minimizing our use of ammunition.

Some specific discussion of the signal processor's job is given in section II below, and a few examples of the results from a simple optimal signal interpreter are given. Then, in section III, some questions are raised which we feel merit serious study.

II. PROBABILITY, LIKELIHOOD RATIOS, AND RELATED TOPICS

The signal processor's proper job lies in the gap between data and decision. His function is to distill from the data the best possible probabilistic description of the current situation. In order to do this, the signal processor must know how the statistics of the received data should vary depending upon the situation--for instance, in a typical case, he must know the statistical description of noise on the one hand and of noise plus target on the other.

Strictly speaking, if the signal processor is to provide a probabilistic interpretation of a message, he must have one other input, the a priori probabilities which describe the pre-message situation.

Let us consider the simplest sort of example. Suppose we are dealing with a two-alternative problem. Suppose the a priori probability of event #1 is α_1 and the a priori probability of event #2 is $\alpha_2 = 1 - \alpha_1$. Suppose we receive a message X. (X may be a number, a telegram, a matrix, or anything.) Let β_1 be the probability that we receive message X if alternative #1 is true. Let β_2 be the probability of receiving message X if alternative #2 is true. Then, the message X tells us that the probability that alternative #1 is the case is

$$\rho_1 = \frac{\alpha_1\beta_1}{\alpha_1\beta_1 + \alpha_2\beta_2} \quad ,$$

and for alternative #2 the probability is

$$\rho_2 = \frac{\alpha_2\beta_2}{\alpha_1\beta_1 + \alpha_2\beta_2} \quad .$$

Many users of data processing outputs are loath to state a priori probability values for the data processors. This is understandable, but the result is that, without α's, ρ's cannot be computed. [Ask a hypothetical question and you get a hypothetical answer.] Actually, any user of signal processing must have--if not specific a priori probabilities in mind--at least a range or zone of values somewhere between 0 and 1 in mind. Otherwise, reflection shows he would have no need for signal processing.

Now, in the two-alternative case at hand, this problem may be neatly circumvented. A little algebra shows that

$$\frac{\beta_1}{\beta_2} = \left(\frac{\rho_1}{1 - \rho_1}\right)\left(\frac{1 - \alpha_1}{\alpha_1}\right) ,$$

so that the effect of the likelihood ratio $\frac{\beta_1}{\beta_2}$ upon the whole family of possible α values can be simply graphed, since the logarithms of the quantities shown are linearly related. Figure I shows such a graph, and shows how a given likelihood ratio transforms any α into the resulting ρ. The diagonal lines are labeled with logs (to the base 10) of the likelihood ratios. Note that sequential independent messages may be treated as a single combined message by multiplying together the likelihood ratios involved. For instance,

suppose that we conduct two independent experiments or receive two independent messages. Suppose that the likelihood ratio resulting from the first is 100, and that from the second is 10. The first experiment converts the a priori probability of .02 to .67 (see chart). Entering with our revised opinion of .67 as the new α, the second experiment (which produces the likelihood ratio 10) converts .67 to .95 which (happily) we note--on further scrutiny of the chart--is the same answer we would have gotten directly by going from .02 to the line for likelihood equals 1000.

As one might expect, things get more complicated when we set out to consider situations with more than two alternatives. A certain amount of geometric visualization is of use here. Note that, in the two-alternative case we have been considering, the universe consists of the straight line interval connecting the points (1,0) and (0,1) in the plane. For the three-alternative problem, the universe is the triangle-plus-its-interior lying in the first octant with vertices at (1,0,0), (0,1,0), and (0,0,1). For the four-alternative case, the universe is the surface-and-interior of the regular tetrahedron with vertices (1,0,0,0) (0,1,0,0) (0,0,1,0) and (0,0,0,1). Etc.

Consider the general case, for the n alternative problem. Once a message is interpreted into a β vector--where for each j, β_j is proportional to the probability that we <u>would have received that message</u> if alternative j were true--then the revised probabilities (ρ_i) are derived from the a priori probabilities (α_i) by the expressions

SUASION CHART

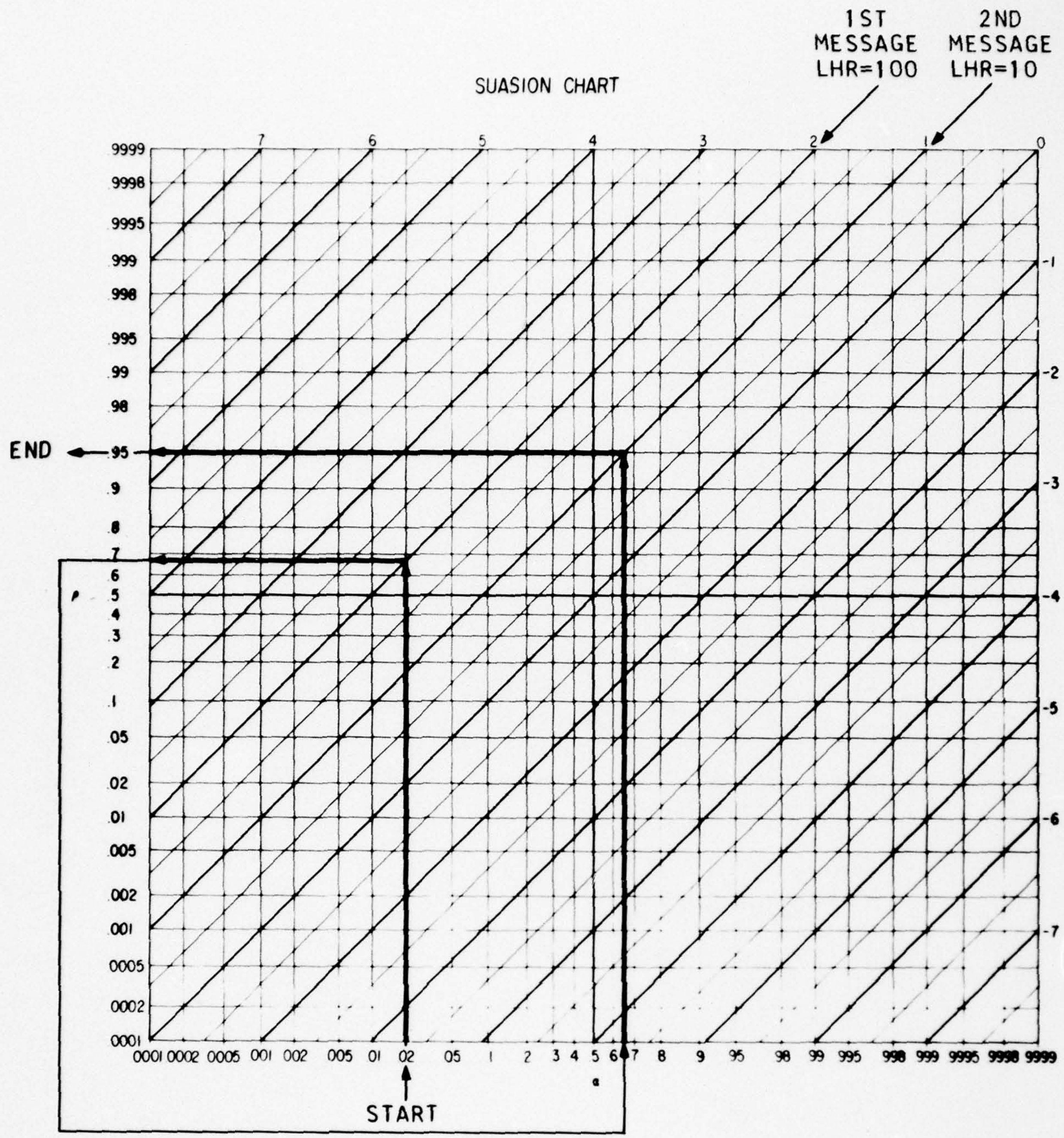

FIGURE 1

$$\rho_j = \frac{\beta_j \alpha_j}{\beta_1\alpha_1 + \beta_2\alpha_2 + \ldots + \beta_n\alpha_n} \quad ,$$

for $\qquad j = 1, 2, 3, \ldots, n \quad .$

Since multiplying each β by the same non-zero scale factor has no effect on the resultant α-to-ρ transformation, it is convenient to rescale the β values such that the sum of the β's is 1. If we adopt this convention, and if we let S_n be the space we have defined for the n alternative problem, then the following simple descriptive remarks are true:

1) The content of any message, once interpreted, is represented by a single point, β, in S_n.

2) β defines a continuous transformation which maps S_n into itself.

3) If β is an <u>interior</u> point of S_n, then the transformation is reversible, and the collection of all such transformations is a commutative group. [Note: The unit element is

$$I_n = \left[\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right]$$

and for an interior point

$$\beta = \left(\beta_1, \beta_2, \ldots, \beta_n\right) \quad ,$$

if we let

$$\Delta = \frac{1}{\beta_1} + \frac{1}{\beta_2} + \ldots + \frac{1}{\beta_n}$$

then the transformation defined by the point

$$\beta' = \left[\frac{\frac{1}{\beta_1}}{\Delta}, \frac{\frac{1}{\beta_2}}{\Delta}, \ldots \right]$$

is the inverse of the transformation defined by the point β.]

4) If β lies on the boundary of S_n--i.e., if one component of β is 0--then the transformation defined by β maps S_n onto a part of its boundary, and reduces the dimension of the problem. This transformation is not reversible. The physical significance is that, in effect, one alternative has been totally ruled out.

Thus, a message is described as a point; and each message defines a mapping of S_n into itself. We may consider each point in its initial position as representing a possible a priori situation, and in its terminal or transformed position as representing the revised probabilities resulting from the message. Figures II-A, II-B, etc., and III-A, III-B, etc., show some examples of a three-alternative problem. The conditions were these:

1) There was one target located at one apex of the triangle [actually in the lower left, but the processor didn't know].

2) The Monte Carlo signals received were exponentially distributed with means N for noise-alone and S + N for for target-plus-noise.

3) The a priori probabilities used to define the starting point were $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ ..., i.e., total uncertainty.

4) The program received successive signals and updated the derived probabilities until within .05 of the correct answer. The value of j shown in each figure is the number of steps required to reach this degree of nearness to the goal.

I have shown a large number of examples to hint at the wide variety of things which can happen. In the words of Mr. J. R. Wright, "He who deals with probability must be prepared to take a chance."

In the multialternative situations there is not in general any simple way to factor the α's and ρ's into separate terms. In fact, the best factoring we can find is of the form

$$\left(\frac{\rho_1}{1 - \rho_1}\right)\left(\frac{1 - \alpha_1}{\alpha_1}\right) = \frac{[\alpha_2 + \alpha_3 + \dots + \alpha_n]\,\beta_1}{\alpha_2\beta_2 + \alpha_3\beta_3 + \dots + \alpha_n\beta_n} \quad .$$

In many cases, the a priori probability distribution is of a special sort. For instance:

If there are n locations where a target might be.

If there is but one target.

If the a priori probability the target is in none of these locations is α.

And if the target-present probability is distributed uniformly over the n locations.

Then we are dealing with an n + 1 alternative problem, thus:

$\alpha_0 = \alpha$ = probability of no target

$$\alpha_1 = \alpha_2 = \ldots = \alpha_n = \frac{1 - \alpha}{n}$$

= a priori probability the target is in the n^{th} location,

so the last expression becomes

$$\left(\frac{\rho_0}{1 - \rho_0}\right)\left(\frac{1 - \alpha}{\alpha}\right) = n\left(\frac{\beta_0}{\beta_1 + \ldots + \beta_n}\right) .$$

This is almost the familiar form which leads to the chart of Figure I, except that we must now use the expression $n\left(\frac{\beta_0}{\beta_1 + \ldots \beta_n}\right)$--a sort of modified likelihood ratio. For example, suppose we are receiving signals from 1000 resolvable locations, so that n is 1001. Suppose that after the signals are processed the normalized $\beta_1, \beta_2, \ldots, \beta_n$ add up to .90909. Then β_0 is .09090, and the virtual likelihood ratio to use is $1000\left(\frac{.0909}{.909}\right) = 100$. This converts the a priori probability of .95 that no target is present into .9994+.

III. QUESTIONS

The essential vulnerability of signal processing to the a priori description may not be avoided--ask a wrong question and you get a wrong answer. The rightness or wrongness of the a priori description, however, does not come within the purview of the signal processor. Another matter is his concern and is a topic of great importance. This is the question "What do errors in the signal processor's notion of the signal probability distributions do to his interpretations?" This very broad question gives rise to a number of specific ones of considerable interest, a few of which are listed below:

1) In a typical signal processing situation, the forms of the distributions of signal plus noise and of noise alone are known, but the values of some of the defining parameters may be known only approximately. For instance, suppose the signal plus noise is Rayleigh distributed in voltage amplitude (or exponentially distributed in power) while the noise alone is also exponentially distributed in power. Then when we receive a single echo power of x, the likelihood ratio to use is obviously

$$\frac{N}{S + N}\, e^{\frac{Sx}{N(S + N)}} \quad .$$

But what happens if we do not know the precise value of S to use? If we use an S which is too big, obviously this will cause us to overrate the

importance of the received signal x. If we use too small an S, the converse will hold. Qualitatively we can see a reassuring trend toward stability here, since if we use too big an S the actual x's we get will tend to be small in comparison to those we would have gotten had the S been correct--so that the overemphasis will be an overemphasis of rather understated signals. The real challenge is to put this matter in quantitative terms. Work needs to be done to determine the sensitivity of such an interpreter to errors in our assessment of S.

2) A completely similar exercise needs to be carried out for the Rayleigh noise and Rayleigh-Rice target situation.

3) The following proposition needs to be investigated--it may be a theorem. "If d and f are the true signal-plus-noise and noise-alone distributions describing a message situation, and if d' and f' are erroneous descriptions used by the signal processor in interpreting the message, then there exists a nonreversible transformation $T(x) = y$ such that if x is distributed according to d, y is distributed according to d' and such that if x is distributed according to f, y is distributed according to f'." If this theorem is true, then errors in interpretation arising from mistaken concepts of d and f may be thought of in the same light as are errors produced by irreversible and

information destroying distortions of the received wave forms.

In order to carry out a systematic investigation of these and related questions, we need to agree upon a general measure of the meaningfulness of an interpreted message. As a candidate I propose the following measure:

A. For the n alternative problem in which we know that there is one and only one target present and in which, after we have interpreted the messages available to us as best we can, our estimate that the target is in the j^{th} location is ρ_j, I propose the measure

$$Q = e^{\sum_{j=1}^{n} \rho_j \ln \frac{1}{\rho_j}} .$$

Note that if $\rho_j \equiv \frac{1}{n}$, $Q = n$. If one of the ρ_j is 1, and the rest are 0, then (in the limit) $Q = 1$. Q stands for quandary. It is a measure related to how much more we would need to know to pin down the target location completely. An optimal processor reduces Q as much as possible. Other processors less so. Therefore, Q is a good measure of the goodness of a processor.

B. For the n + 1 alternative case where the extra alternative covers the possibility the target is not there at all, the problem is more complex. One alternative would be to define Q as before; but I feel the need to segregate

the target-not-present case from the target-present-but-in-an-unknown-location case. One possibility is to re-normalize the target present probabilities thus

$$\rho_j' = \frac{\rho_j}{\sum_{\ell=1}^{n} \rho_\ell} \quad ,$$

and then let

$$U = (1-\rho_0) \times \sum_{j=1}^{n} \rho_j' \ln \frac{1}{\rho_j'}$$

$$\cancel{U = (1 - \rho_0)}\ e \quad ,$$

where ρ_0 is the derived probability the target is absent.

Note that when we know the target is present, U = Q, and that otherwise U is the probability that a target is present <u>times</u> our quandary regarding where it is. U stands for urgency.

It is my hope and belief that these definitions will prove heuristic in our investigation of the interpretation problems, and will lead to useful results directly applicable to systems planning.

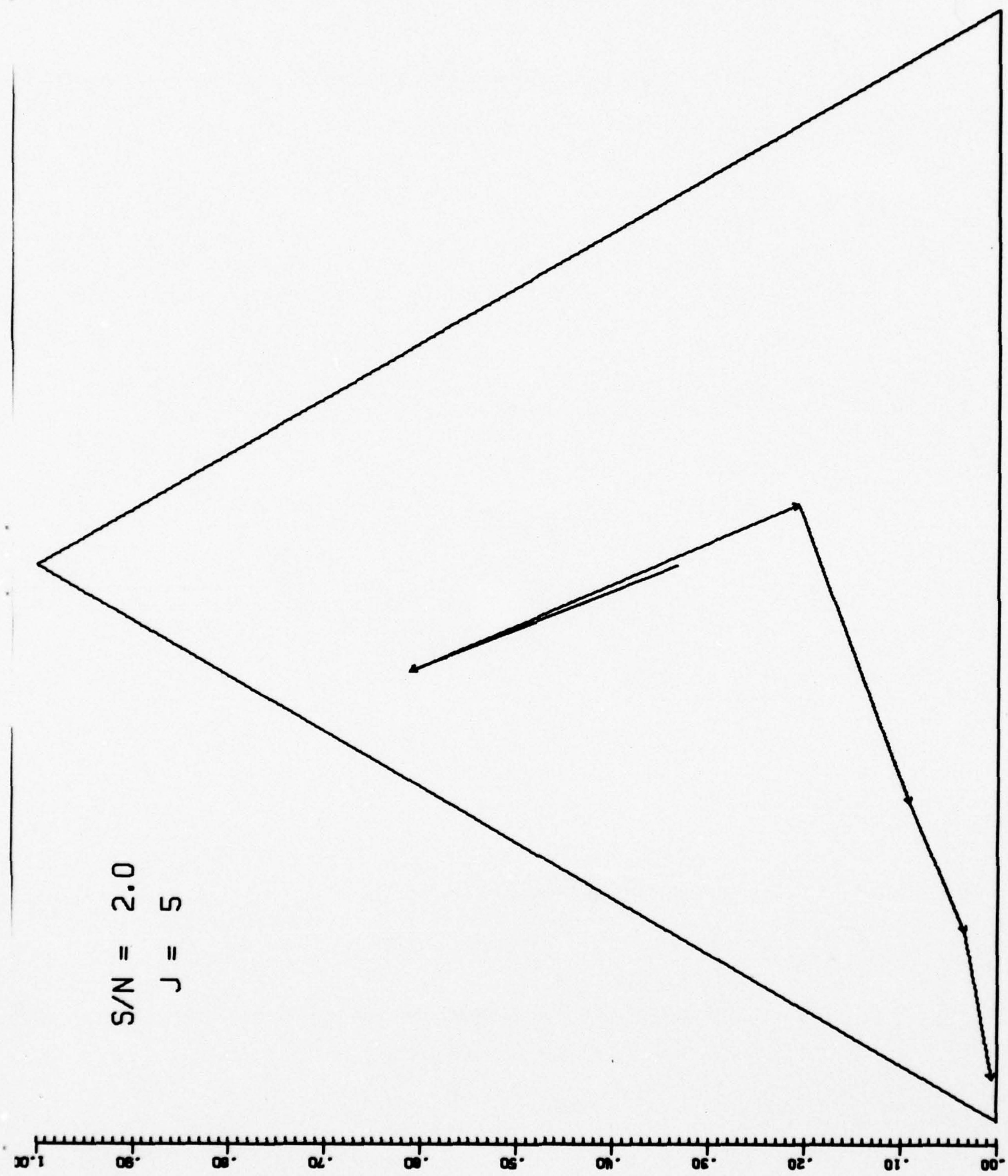

FIGURE II-A

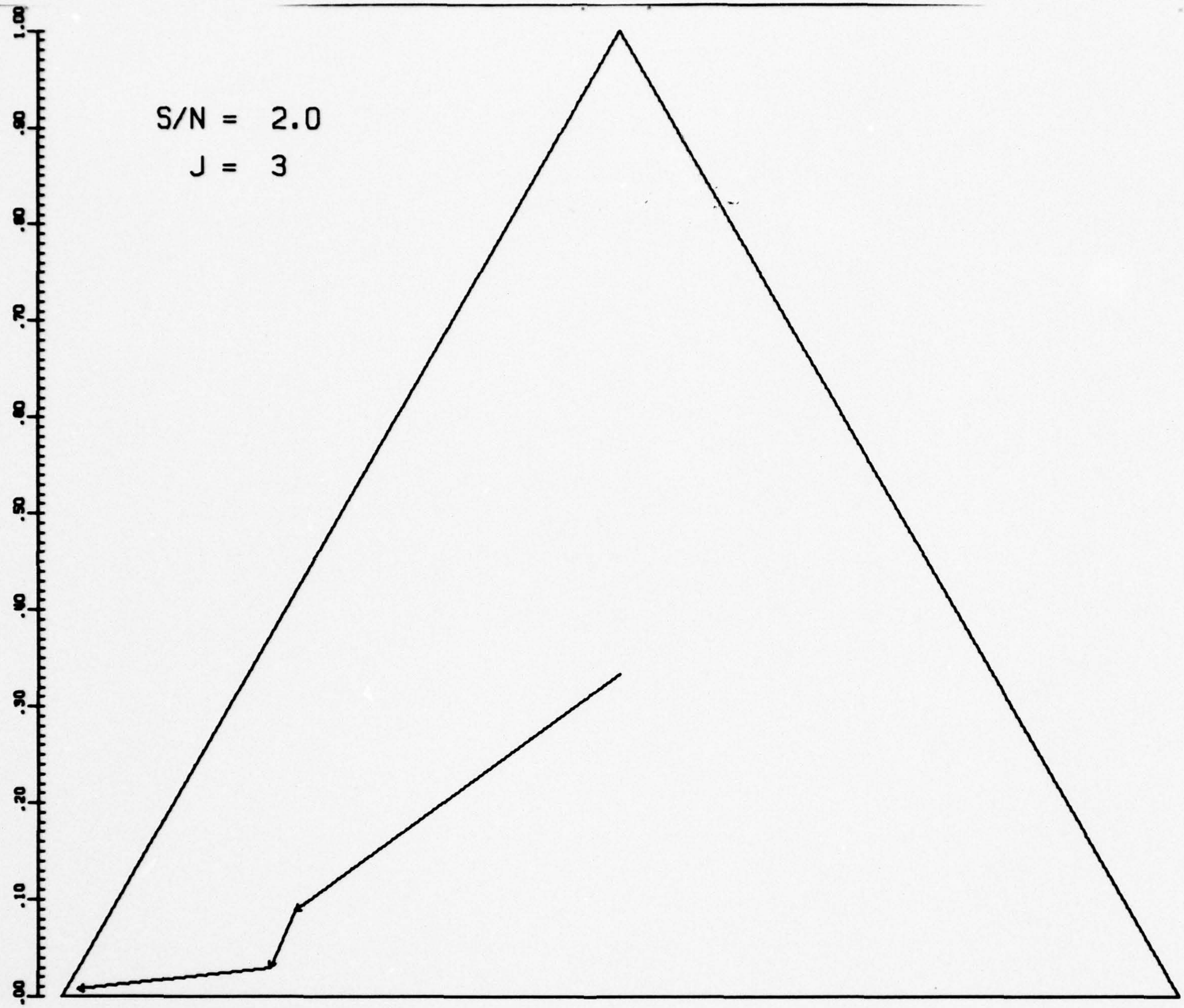

FIGURE II-B

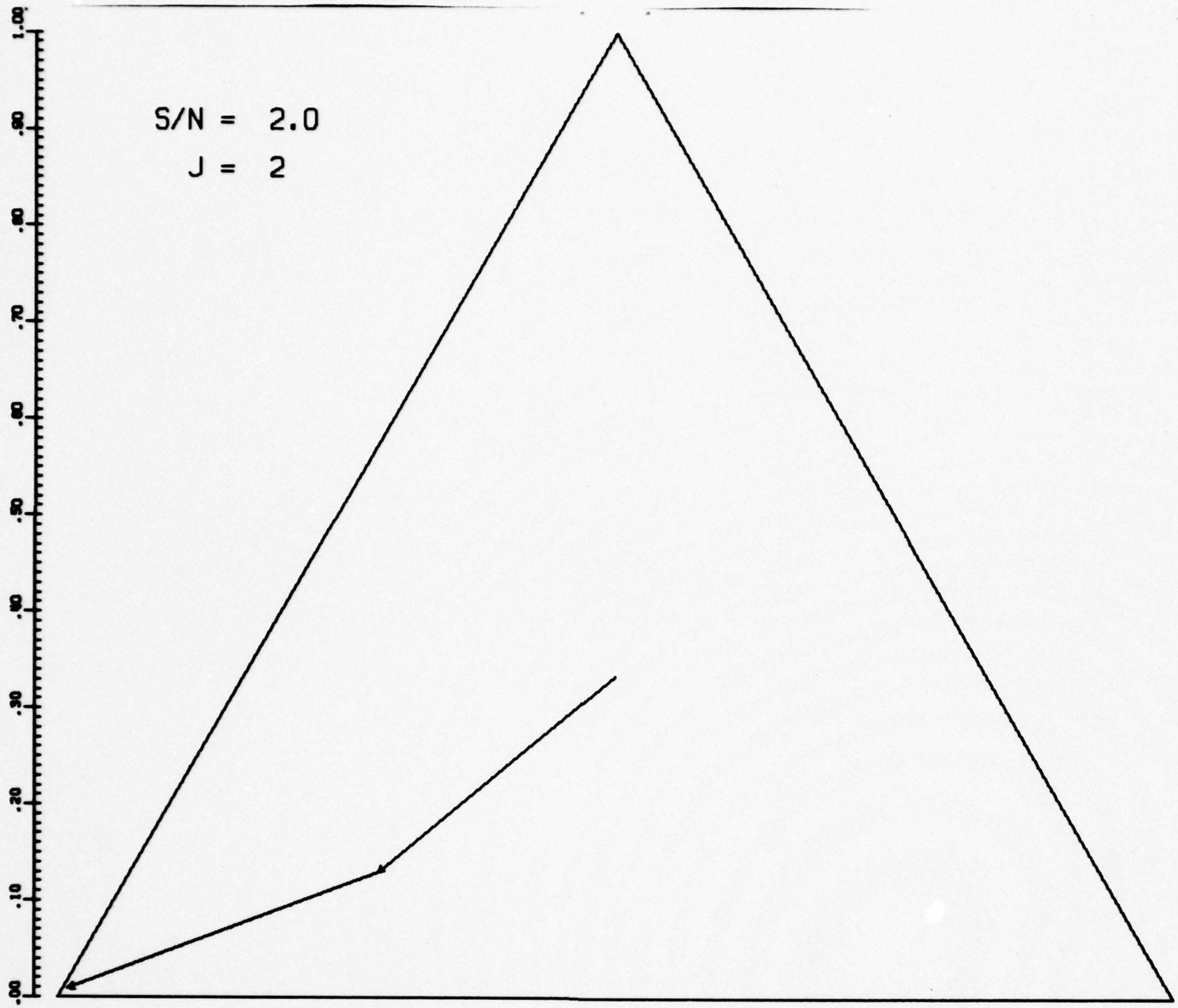

FIGURE II-C

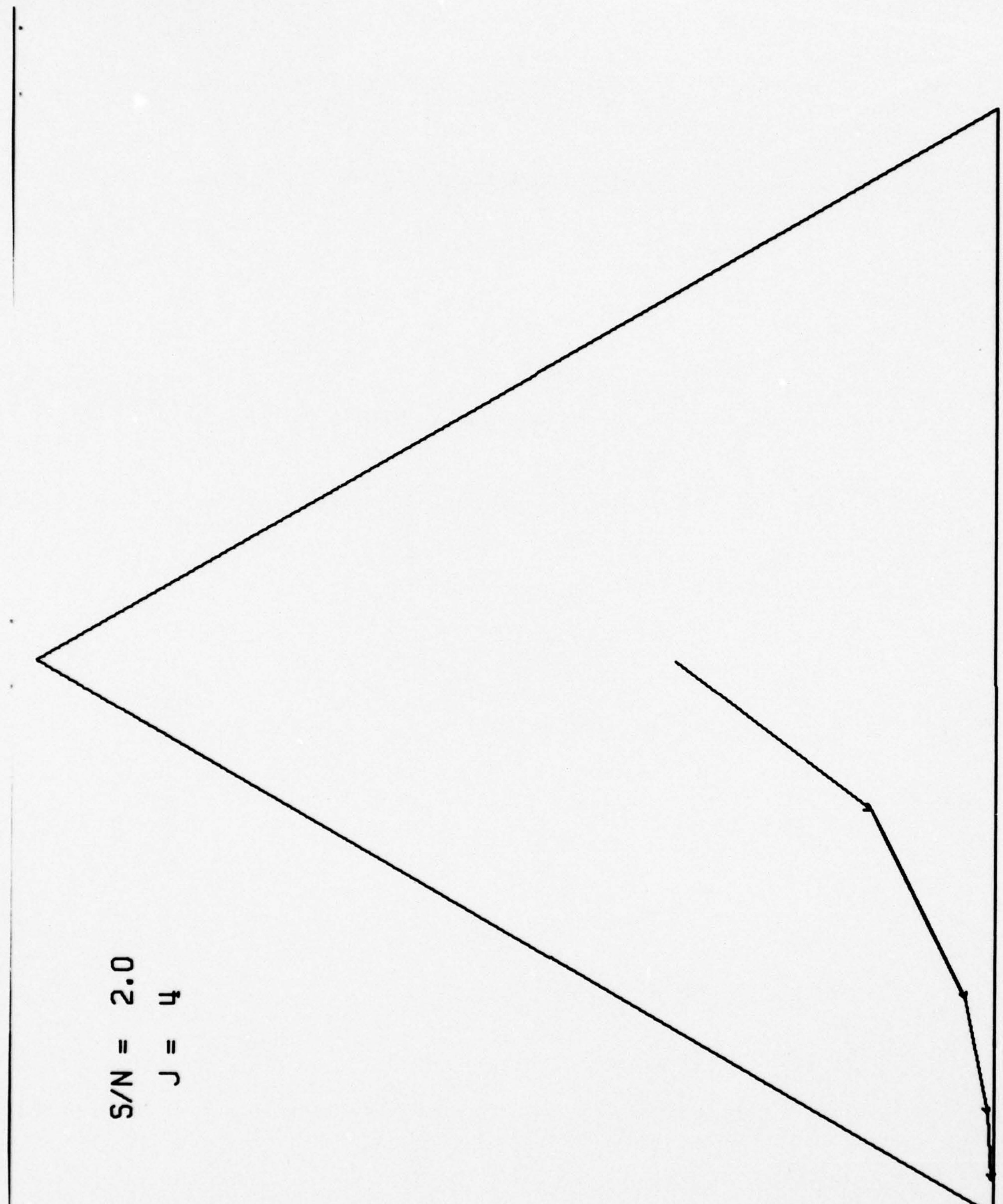

FIGURE II-D

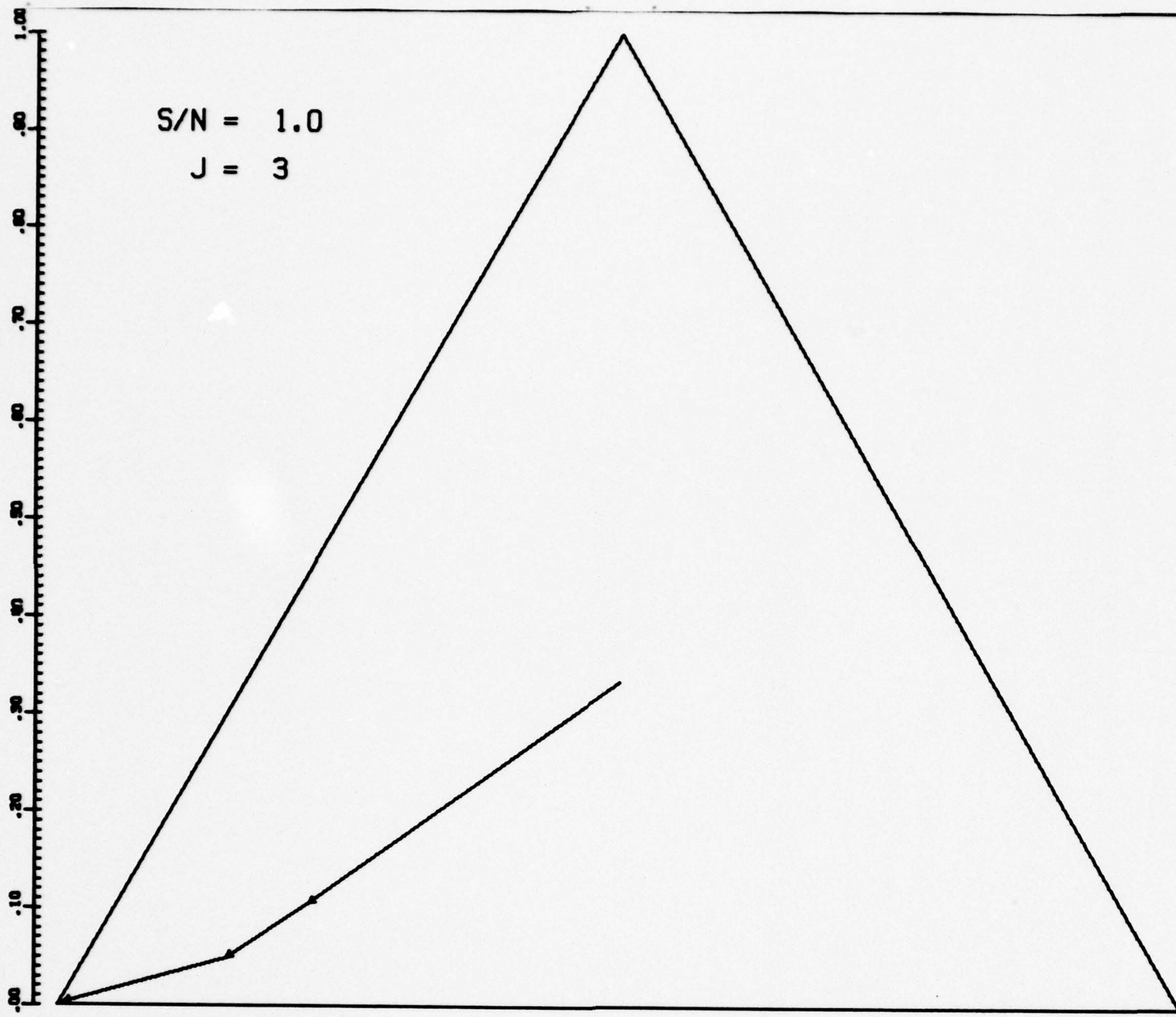

FIGURE III-A

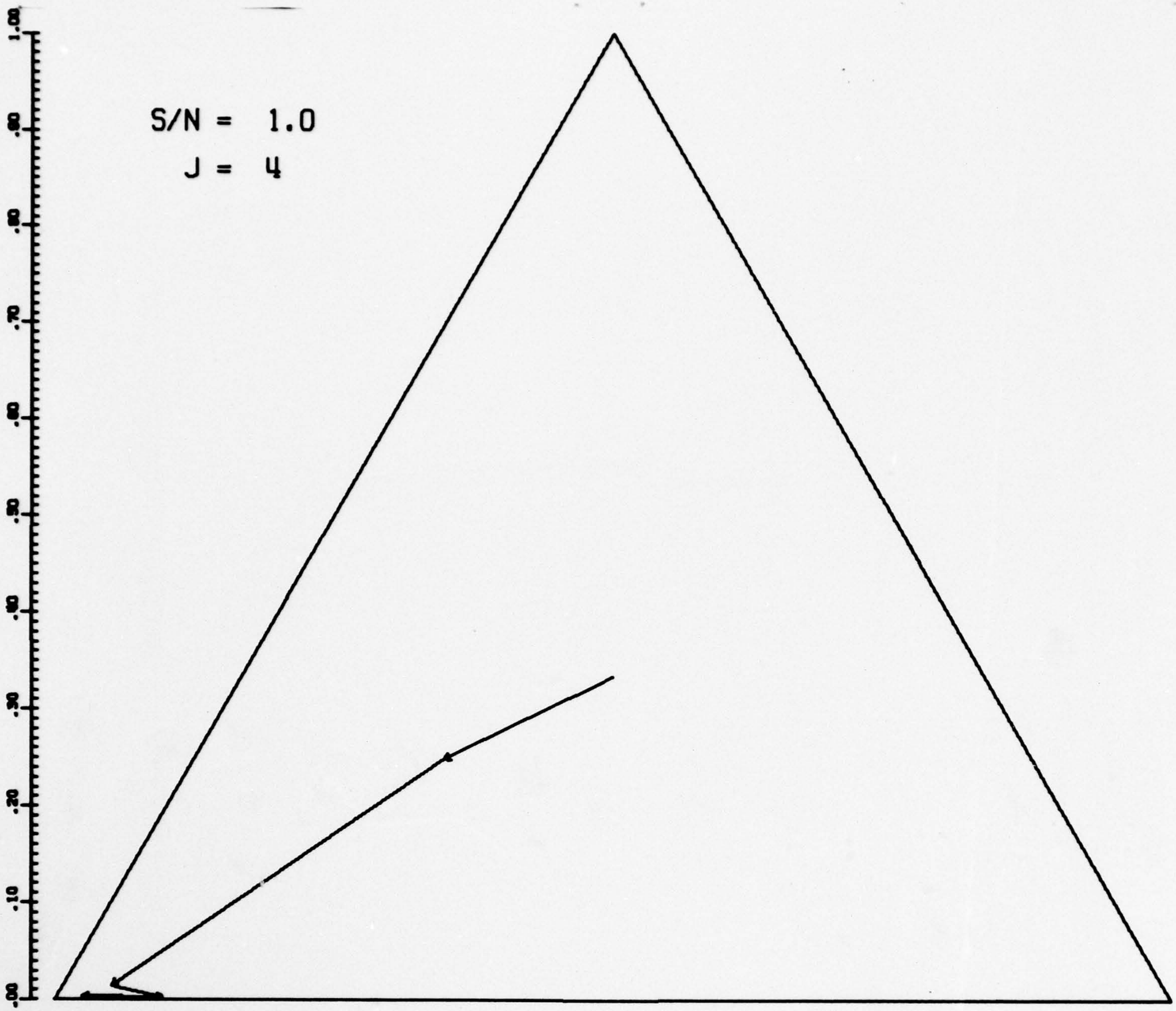

FIGURE III-B

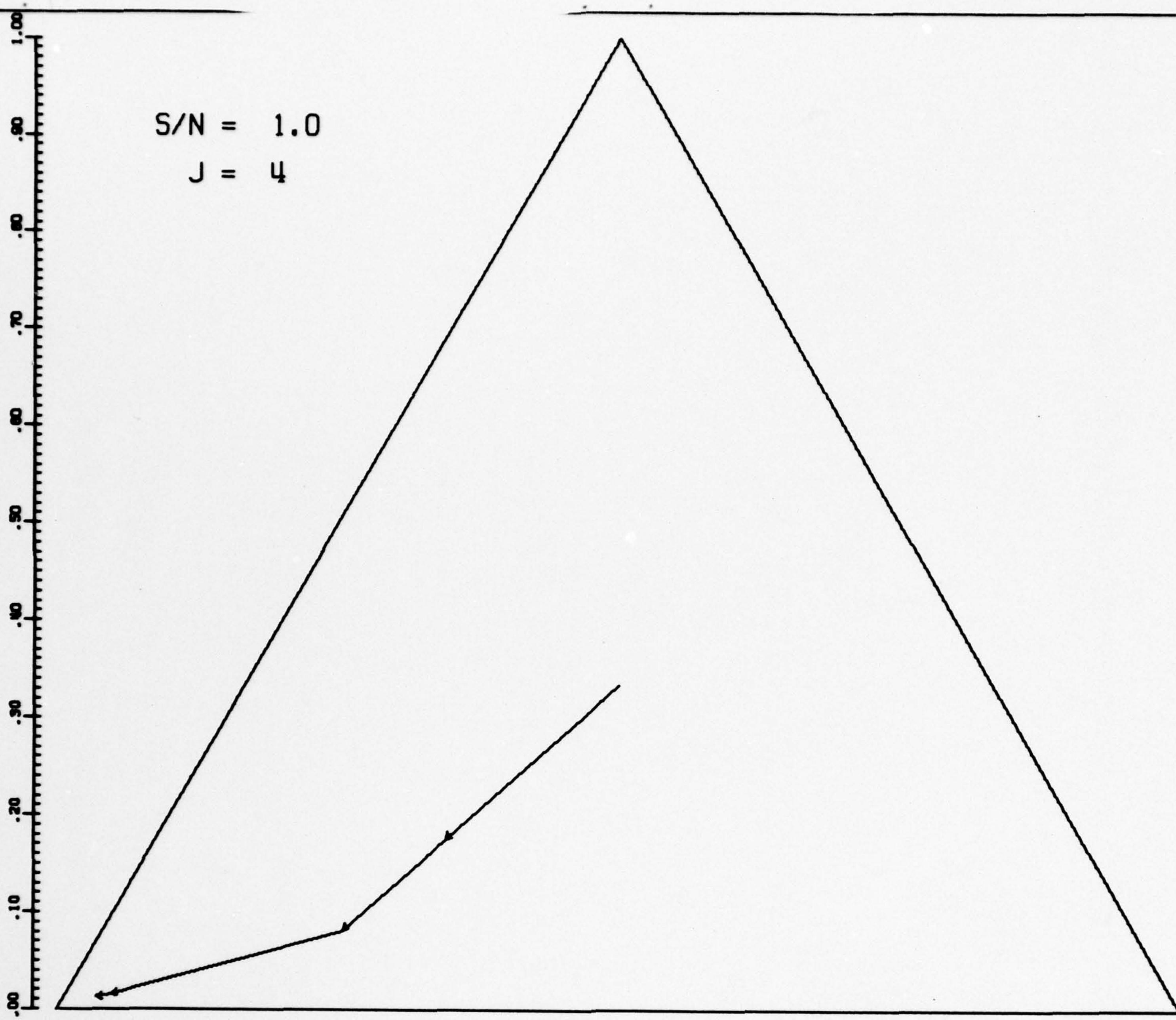

FIGURE III-C

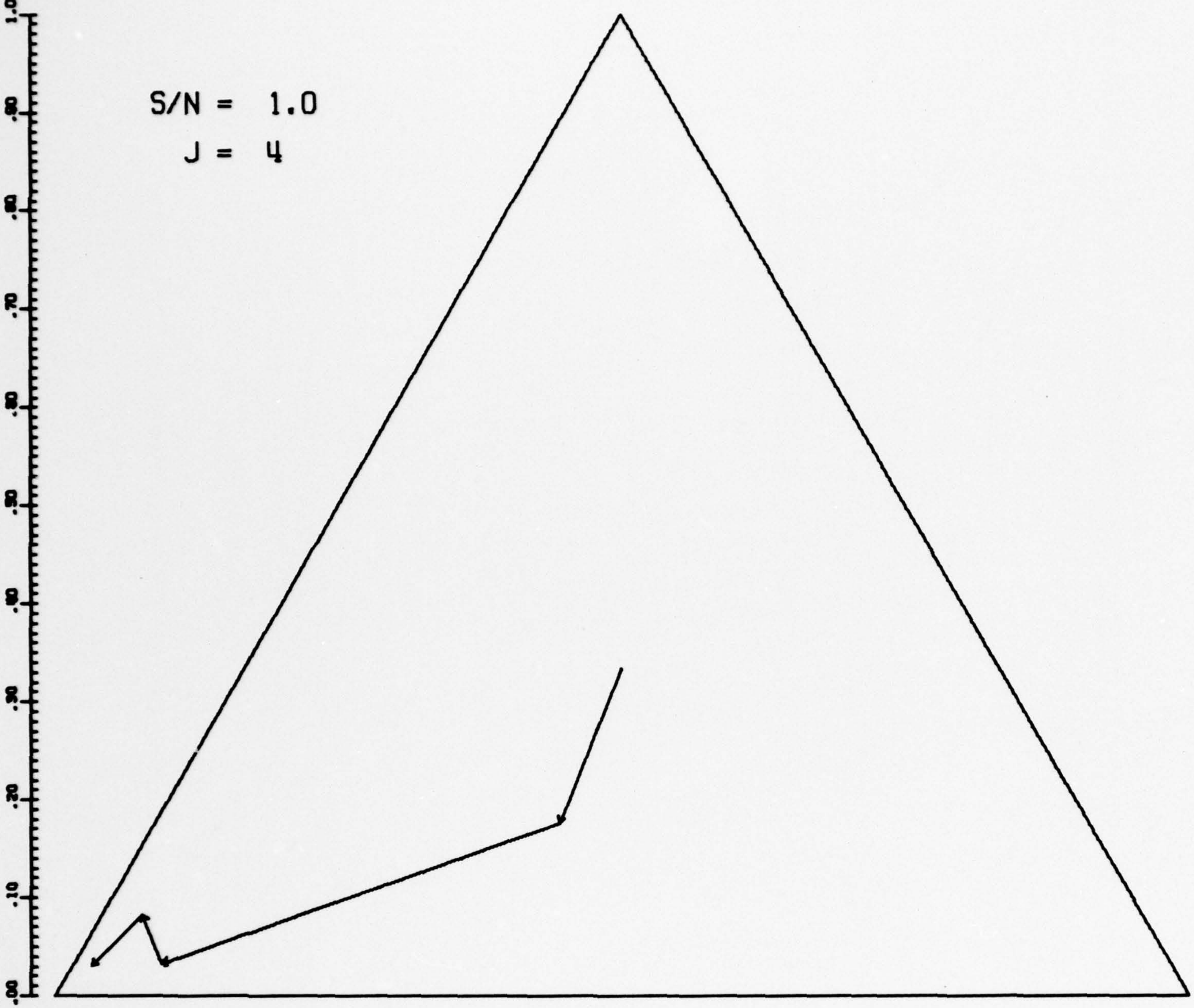

FIGURE III-D

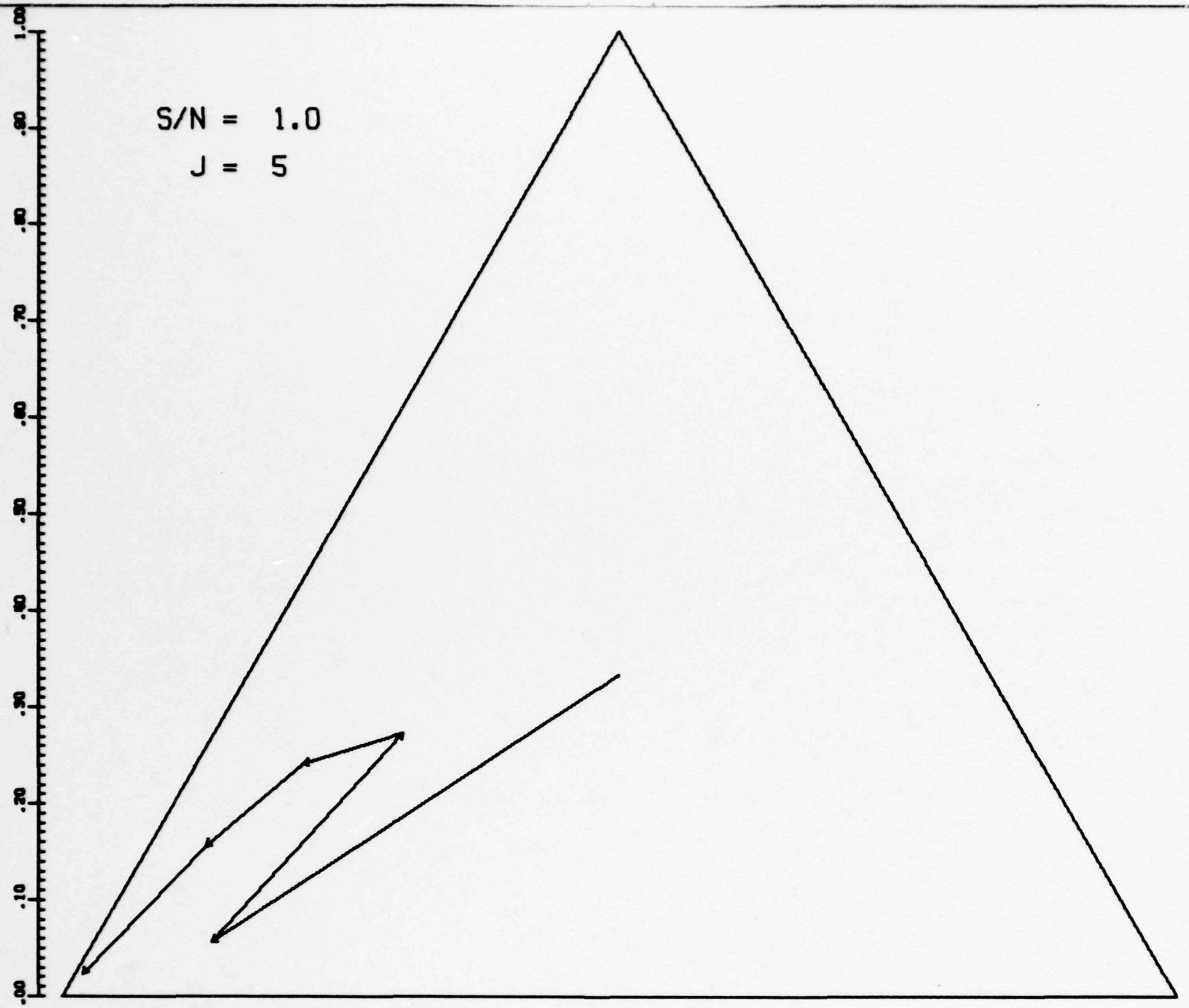

FIGURE III-E

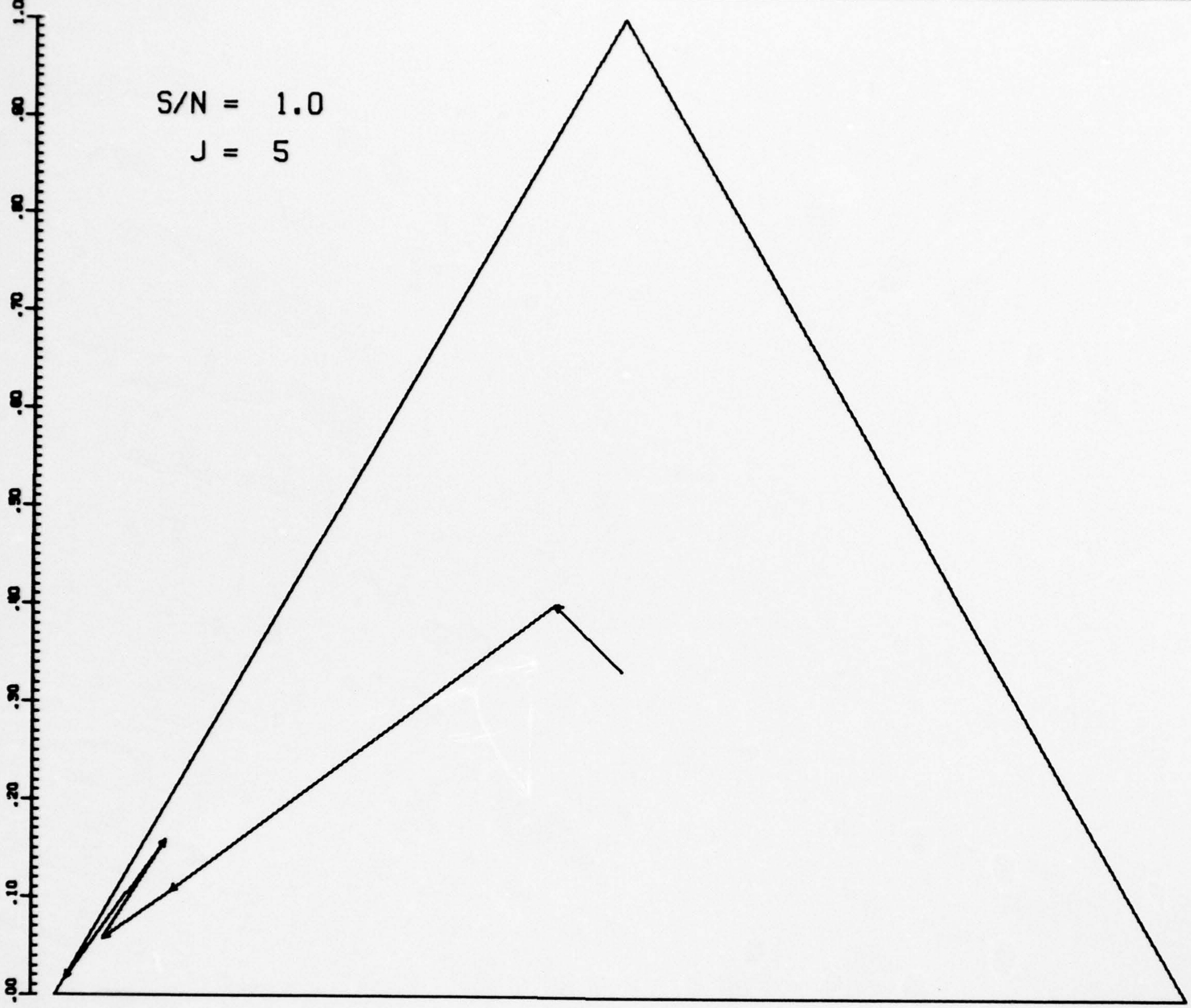

FIGURE III-F

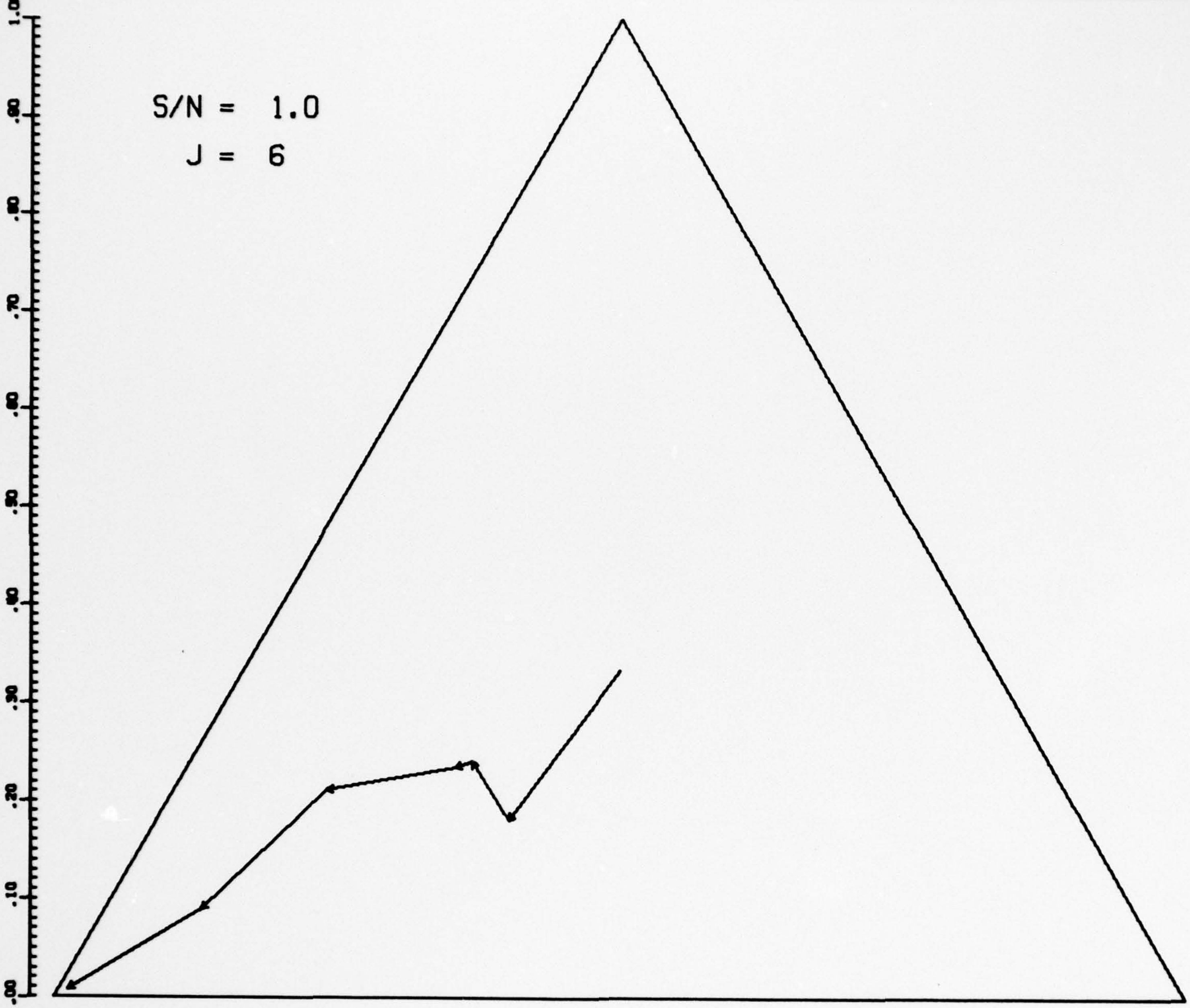

FIGURE III-G

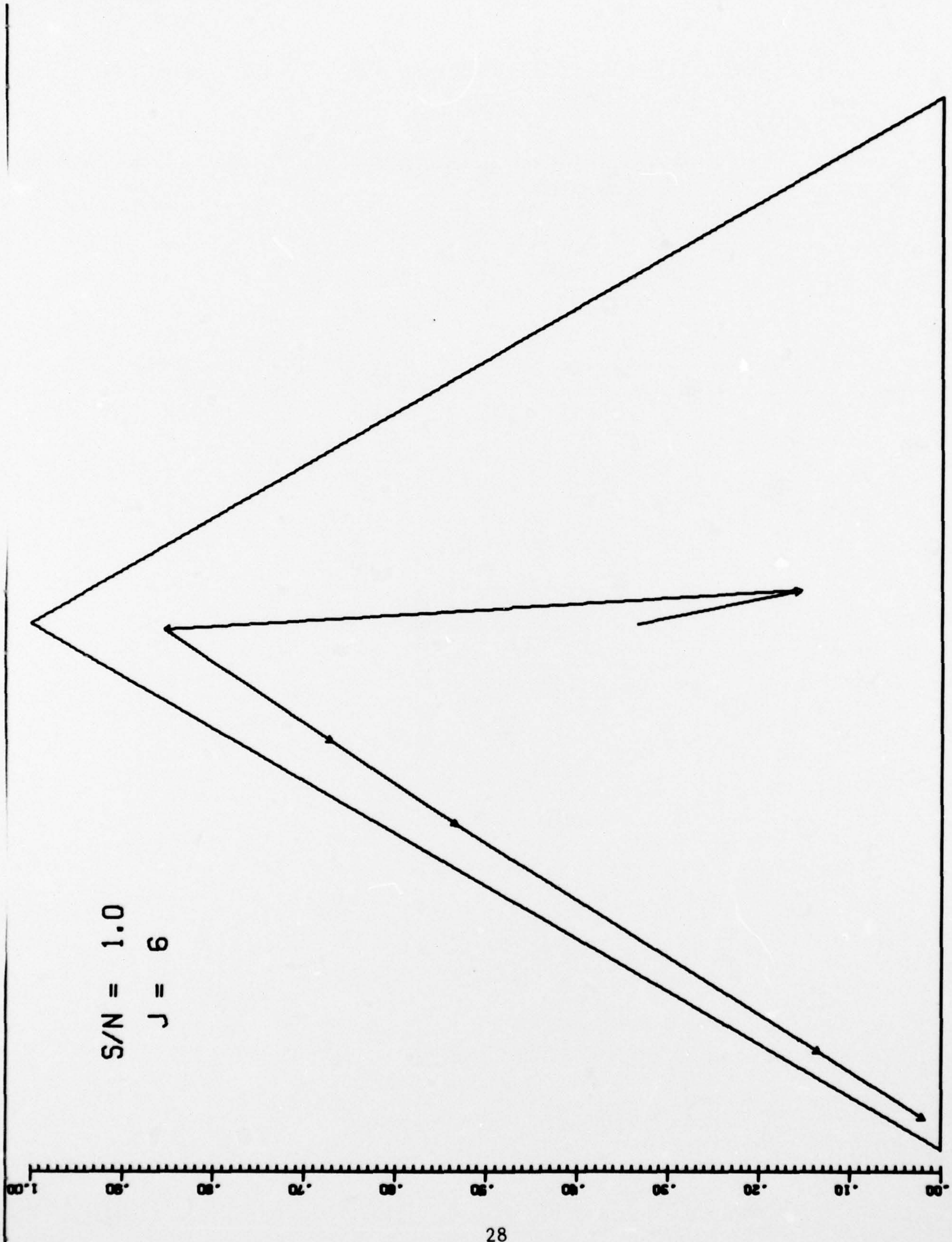

FIGURE III-H

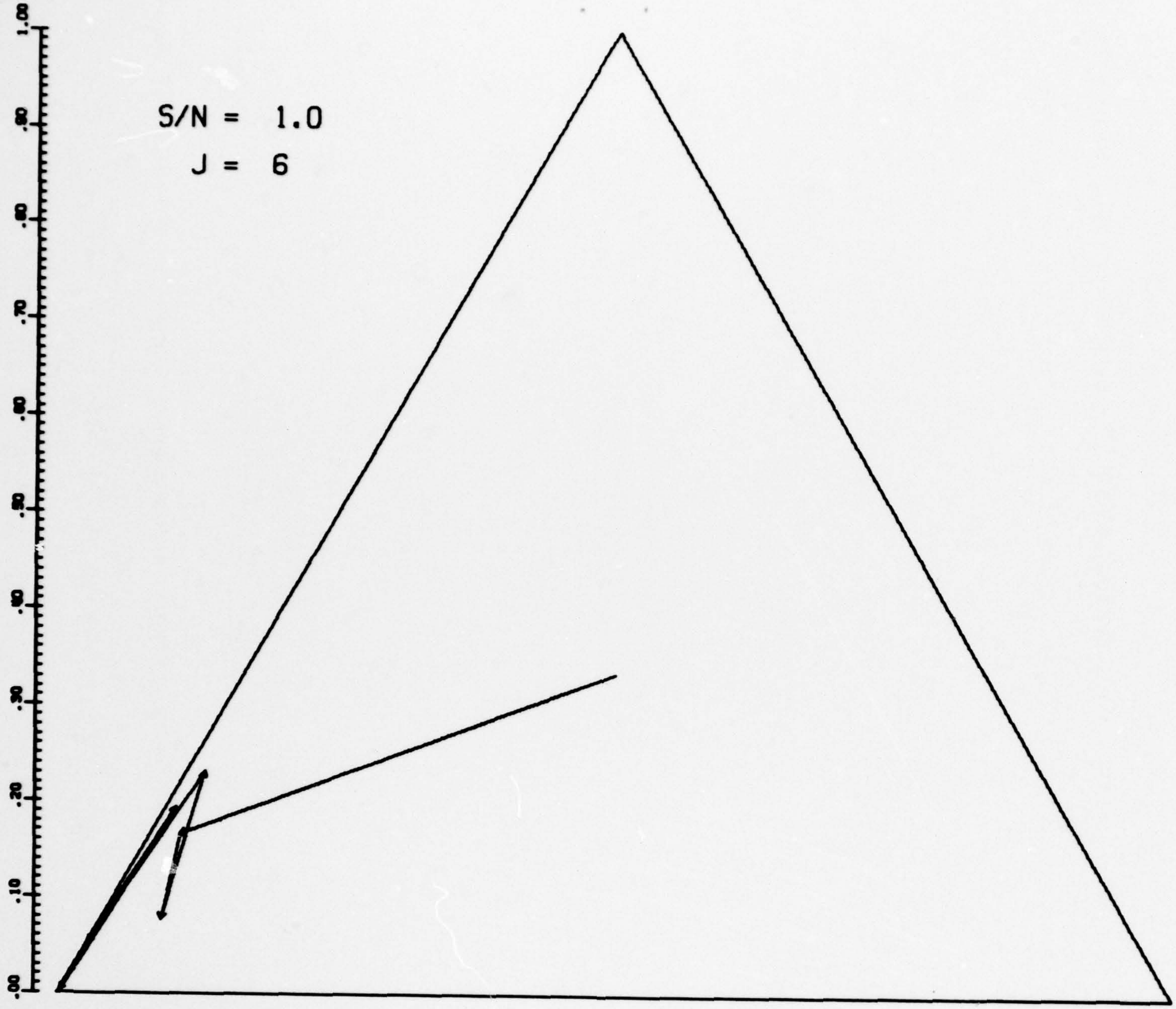

FIGURE III-J

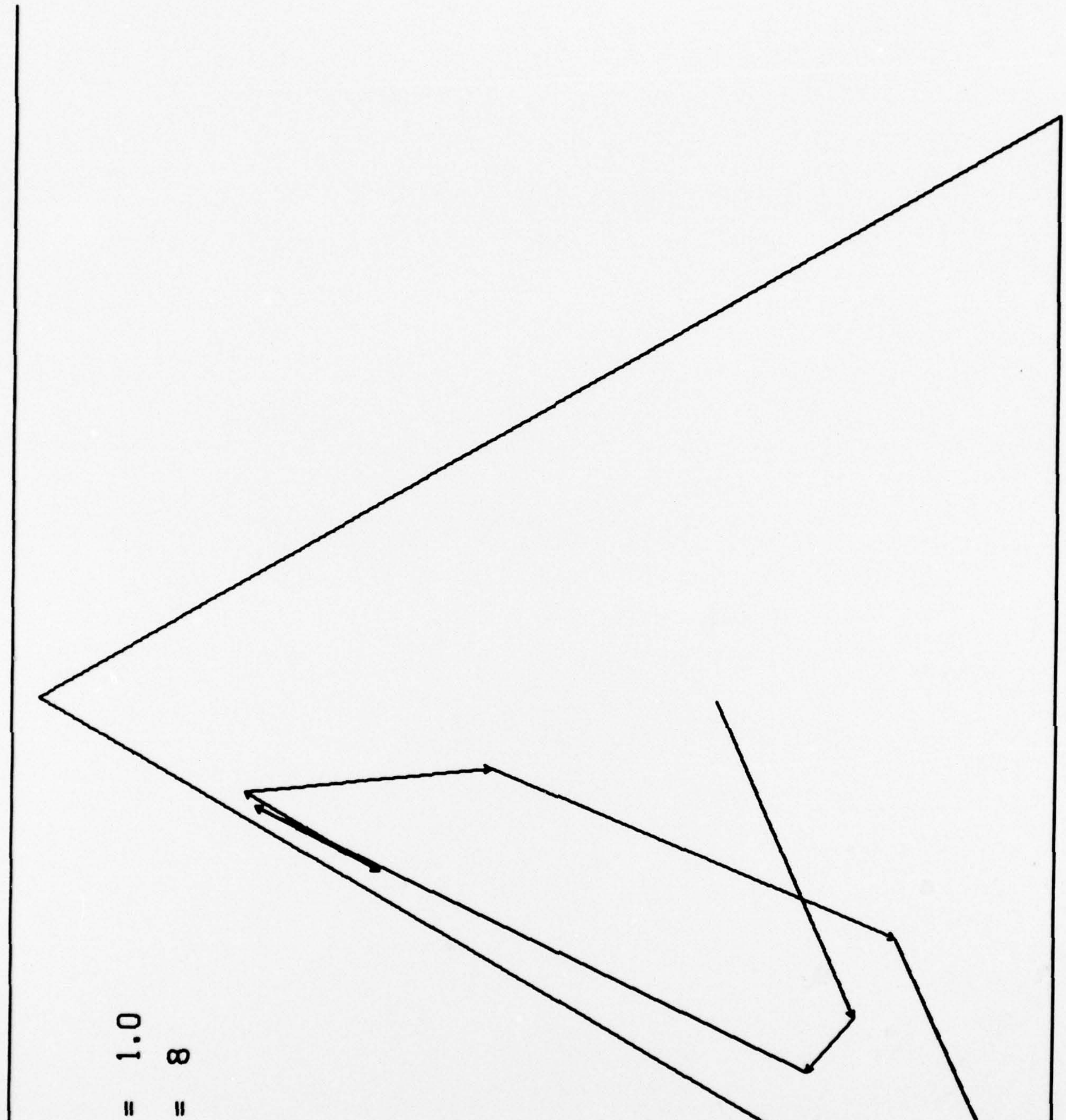

FIGURE III-K

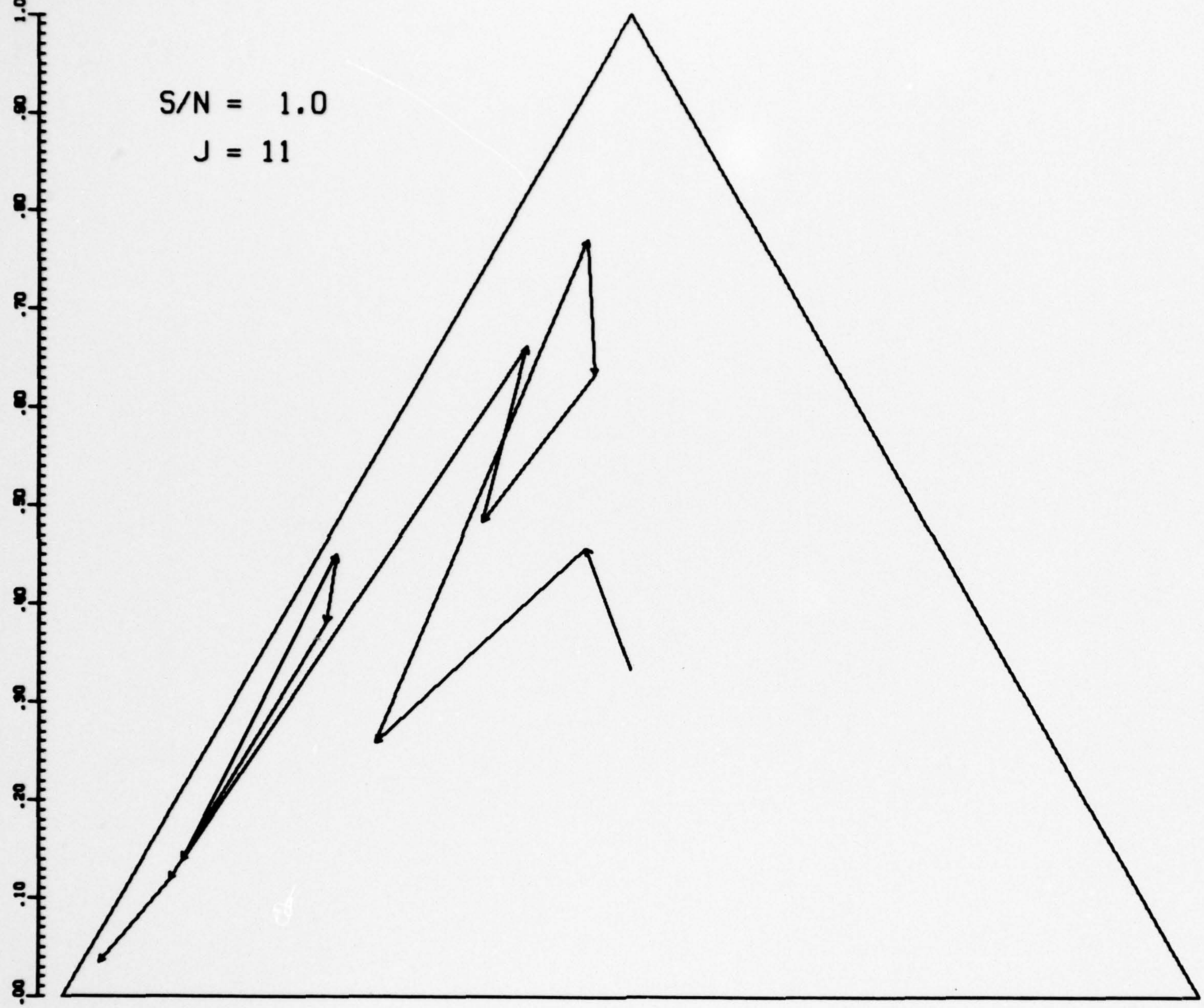

FIGURE III-L

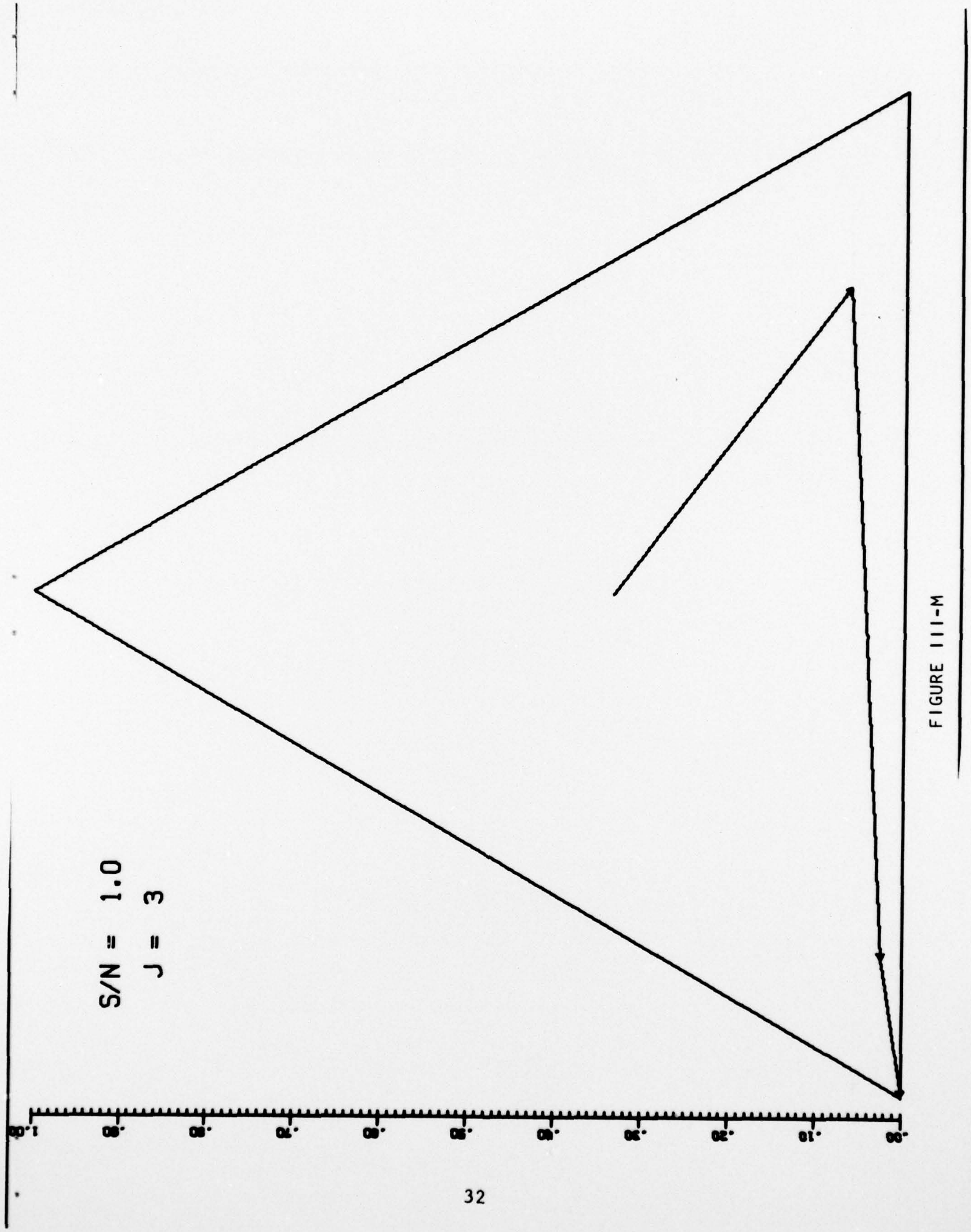

FIGURE III-M

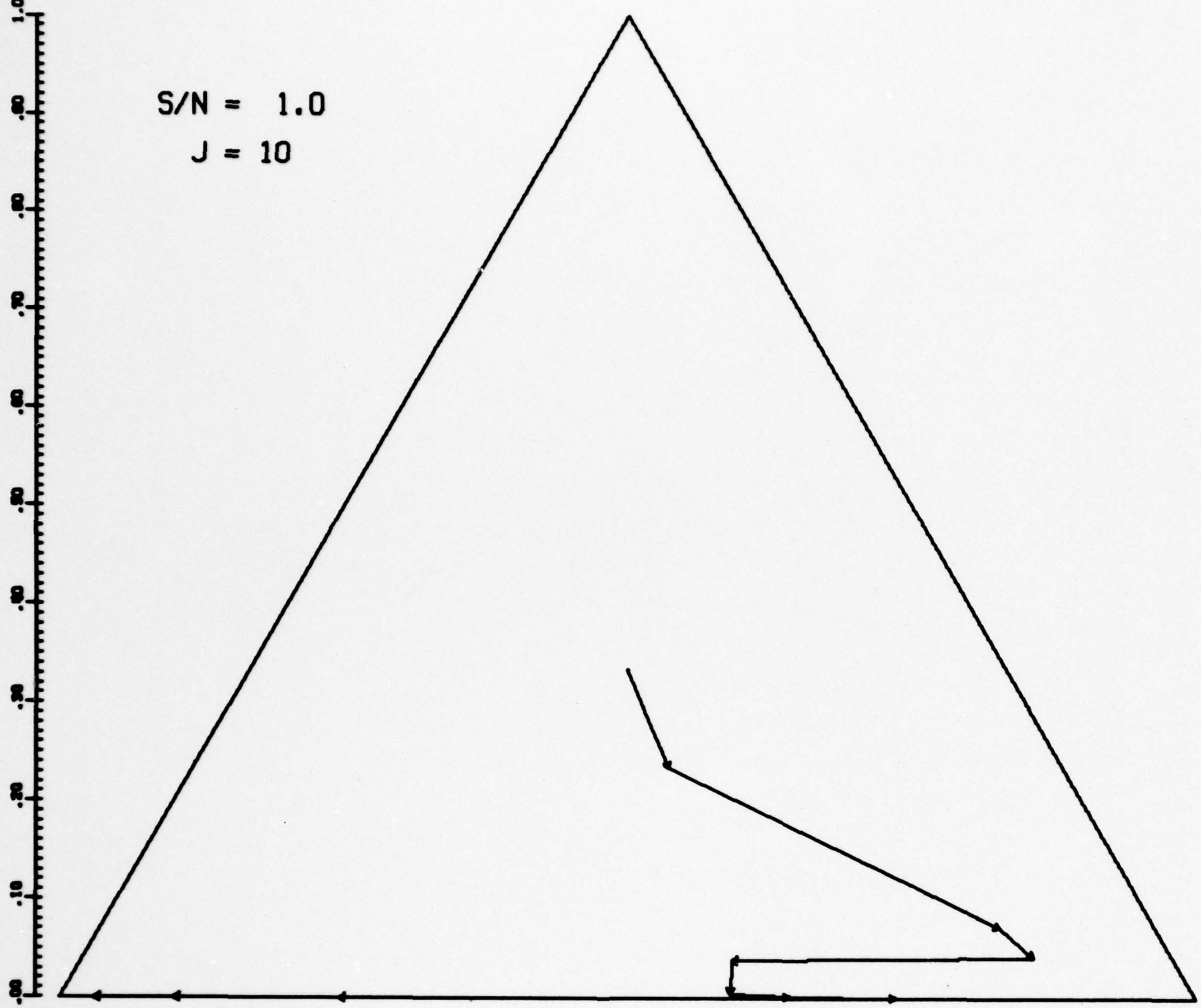

FIGURE III-N

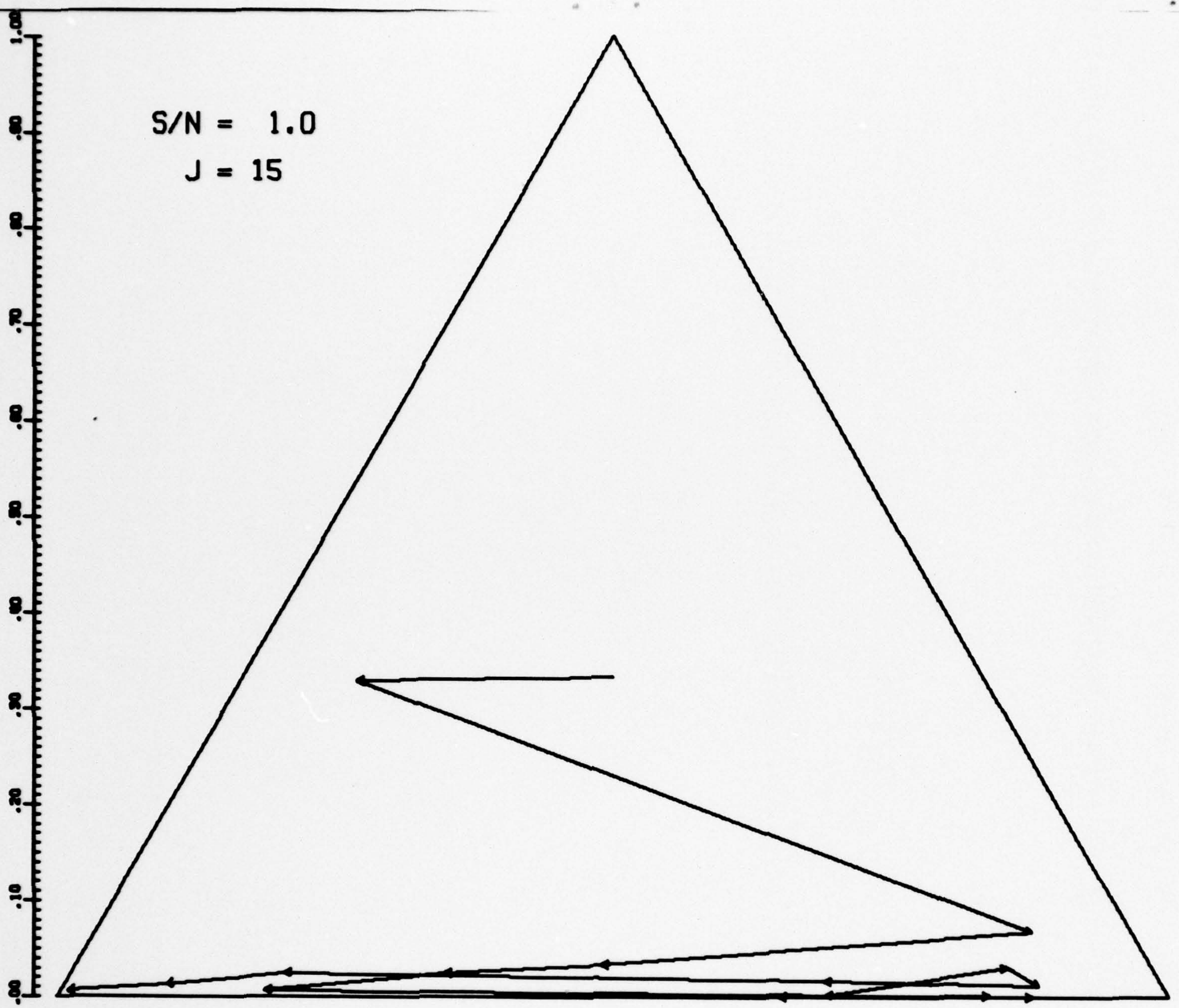

FIGURE III-0

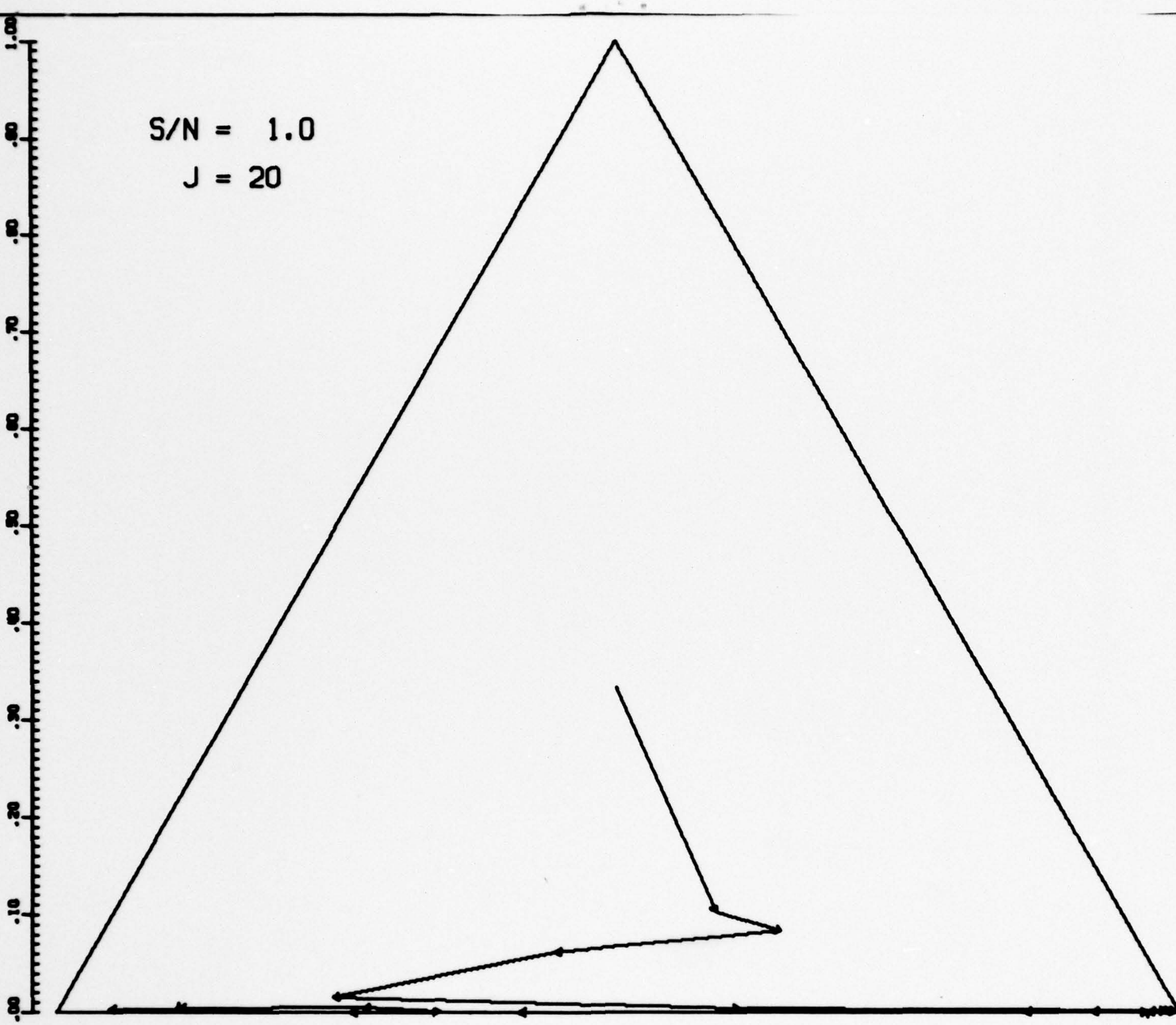

FIGURE III-P

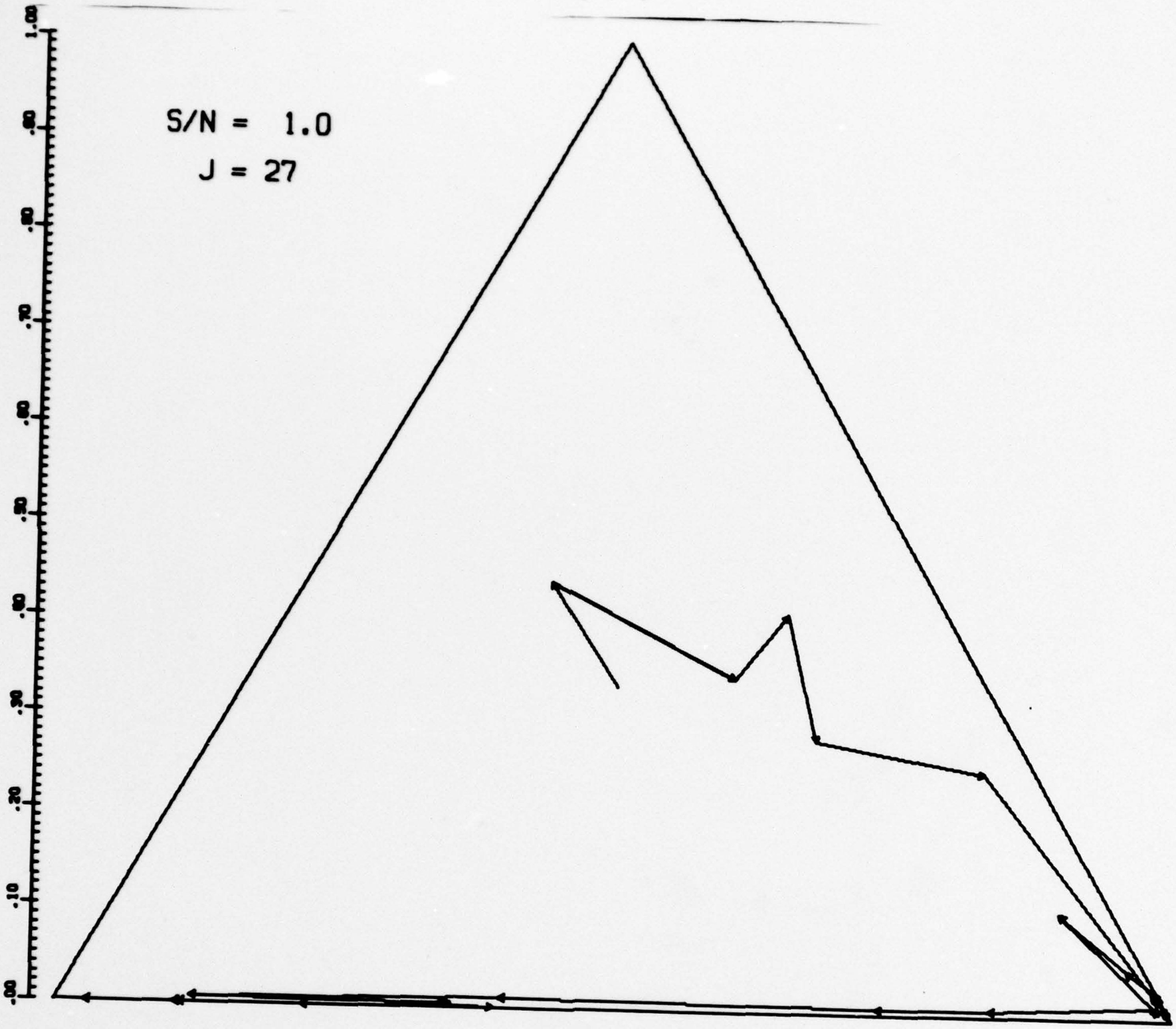

FIGURE III-Q